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Penalty-Function Guidance for Multiple-Satellite Cluster Formation

Hiroaki Umehara*
University of Glasgow,
Glasgow, Scotland G12 8QQ, United Kingdom
and
Colin R. McInnes†
University of Strathclyde,
Glasgow, Scotland G1 1XJ, United Kingdom

Introduction

DISTRIBUTED systems with a cluster of multiple artificial satellites have been considered to be beneficial for telecommunications[1,2] and other science missions.[3] Therefore, advanced operations of a cluster formation have a crucial role on such space applications. Clustering of two satellites has already been studied by analyzing just one relative orbit of the Clohessy–Wiltshire (CW) rotating coordinate frame (see Ref. 4). Near-miss avoidance imposes more complex constraints as a cluster is composed of more satellites. It is probable that one satellite escaping from another satellite encounters other satellites if near-miss avoidance strategy between all of the satellites in close proximity is not fully considered. One solution is to accelerate the respective satellites as if they have virtual repulsion. Such potential fields have been formulated and imposed on the state-variable space to form an equidistant constellation around the Earth[5] and a cluster center,[6] respectively. Other solutions have been also proposed: graph theory for switching the roles of leaders and followers[7] and mixed-integer linear programming.[8]

However, guidance by a potential field is still important because fast computation is possible even in the case with unspecified multiple near misses by many satellites. The problem is that finding a potential function to decrease fuel consumption is difficult, which motivates us to introduce a near-miss avoidance potential in the costate-variable space. This Note solves the fuel-optimization problem with state constraints by a penalty-function method.[9] The disadvantage of using the costate variables is that an iterative algorithm is inevitable to tackle the two-point boundary-value problem. Such a difficult computation problem can be overcome by appropriate selection of function parameters, which is the final topic of this Note.

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*Visiting Research Fellow, Department of Aerospace Engineering; also Senior Researcher, Space Cybernetics Group, National Institute of Information and Communications Technology, Kashima Space Research Center, Hirai, Kashima 314-8501, Japan; ume@nict.go.jp.

†Professor, Department of Mechanical Engineering; colin.mcinnes@strath.ac.uk. Member AIAA.

Formulation

Consider guidance of eccentricity-inclination separation, a popular method of cluster formation, in three-dimensional space.[10] Orbits of all satellites approaching eccentricity-inclination separation are restricted approximately on a fixed plane passing through the origin in the CW coordinate frame. Some near misses can be avoided orthogonally, whereas the in-plane positions coincide with each other if a satellite configuration is far from eccentricity-inclination separation. However, almost all of the in-plane near misses in a process for eccentricity separation are projected onto three-dimensional near misses near an eccentricity-inclination-separation configuration. Therefore, the dynamics of the planar clustered n-satellite system is considered around an orbiting point on a circular reference orbit.

Each satellite S_i, $i = 1, \ldots, n$, is assumed to have continuous longitudinal thrust. Linearizing the Keplerian orbital dynamics near the reference orbit[11] and then scaling the CW coordinate frame yields

$$\boldsymbol{x}_i' = \boldsymbol{f}_i(\boldsymbol{x}_i, u_i), \qquad \boldsymbol{f}_i = \begin{bmatrix} x_i^{(3)} \\ x_i^{(4)} \\ 3x_i^{(1)} + 4x_i^{(4)} \\ -x_i^{(3)} + u_i \end{bmatrix} \tag{1}$$

with $\boldsymbol{x}_i = (x_i^{(1)}, x_i^{(2)}, x_i^{(3)}, x_i^{(4)})^T$, where $x_i^{(1)}$ and $2x_i^{(2)}$ are the radial and longitudinal coordinates of the ith satellite from the orbiting coordinate frame on the reference orbit, respectively, and $2u_i$ is the thrust acceleration. Note that the longitudinal components, $x_i^{(2)}$ and u_i, are scaled down by half. The unit time is adopted so that the orbital frequency is equal to 1. The time derivatives of $x_i^{(1)}$ and $x_i^{(2)}$ give $x_i^{(3)}$ and $x_i^{(4)}$, respectively. If u_i vanishes, then

$$\begin{bmatrix} x_i^{(1)} \\ x_i^{(2)} \end{bmatrix} = \begin{bmatrix} -\left(\frac{4}{3}\right)\epsilon_i^{(2)} + \epsilon_i^{(3)}\cos t + \epsilon_i^{(4)}\sin t \\ \epsilon_i^{(1)} + \epsilon_i^{(2)}t - \epsilon_i^{(3)}\sin t + \epsilon_i^{(4)}\cos t \end{bmatrix} \tag{2}$$

with in-plane orbital elements $\epsilon_i^{(k)}$, $k = 1, \ldots, 4$. Here, $2\epsilon_i^{(1)}/a$, $2\epsilon_i^{(2)}/a$, and $(\epsilon_i^{(3)}/a, \epsilon_i^{(4)}/a)$ define a nominal longitude, a drift rate, and an eccentricity vector, respectively, where a is the semimajor axis of the reference orbit. If all near misses are avoided, the formulation of the final state corresponding to eccentricity separation simplifies to $\epsilon_i^{(1)} = \epsilon_i^{(2)} = 0$ without constraints on $(\epsilon_i^{(3)}, \epsilon_i^{(4)})$. Therefore, the terminal constraint is

$$\boldsymbol{\psi}_i(t_f) = \boldsymbol{0}, \qquad \boldsymbol{\psi}_i = \left(x_i^{(1)} + x_i^{(4)}, x_i^{(2)} - x_i^{(3)}\right)^T \tag{3}$$

The fuel-consumption index is defined by

$$J = \sum_{i=1}^{n} \int_0^{t_f} \frac{1}{2} u_i^2 \, \mathrm{d}t \tag{4}$$

Note that control of the eccentricity is necessary if a tight cluster is to be formed. For this purpose, a constraint of $[x_i^{(1)}(t_f)]^2 + [x_i^{(2)}(t_f)]^2$ should be added to Eq. (4), which is, however, out of the scope of this Note.

To avoid any near miss and reduce thruster fuel consumption, the minimization of the following function is considered:

$$J^* = J + \sum_{i=1}^{n} \left\{ \boldsymbol{\nu}_i^T \boldsymbol{\psi}_i(t = t_f) + \int_0^{t_f} \left[\sum_{j>i}^{n} w(r_{ij}) + \boldsymbol{\lambda}_i^T (\boldsymbol{f}_i - \boldsymbol{x}_i') \right] \mathrm{d}t \right\} \tag{5}$$

by using the costate variables $\boldsymbol{\lambda}_i(t) \in \mathcal{R}^4$, the Lagrange multipliers $\boldsymbol{\nu}_i \in \mathcal{R}^2$, and the penalty function $w(r_{ij}) \in \mathcal{R}$, where r_{ij} is the distance between S_i and S_j. The initial and the final times are fixed as 0 and t_f, respectively. The minimization of Eq. (5) does not exactly give the fuel-minimized solution. The optimal orbits obtained by minimizing J^* are not necessarily restricted in a domain satisfying $w(r_{ij}) = 0$ even if the range of w includes zero. In this case, the solution is not optimized exactly. Even in the opposite case, the control function solved by the minimization of J^* with $n > 2$ does not necessarily minimize the total fuel consumption for all of the satellites, which is proportional to the total velocity change defined by

$$\mathrm{tot}|\Delta v| = \sum_{i=1}^{n} \int_0^{t_f} |u_i(t)| \, \mathrm{d}t \tag{6}$$

However, the consumption difference from the truly minimum fuel is considered moderate, which will be shown by simulations with several function parameters in the next section.

The near-miss-avoidance penalty function $w(r)$ is selected with the property that $w(0) = \alpha$, $w(r)$ decreases with increasing $|r|$ for $|r| \le \beta$, and $w(r) = 0$ for $|r| \ge \beta$. If $w(r) \to \infty$ as $r \to 0$ is adopted for full safety from collision, the repulsive effect is too strong to search for an optimum initial $\boldsymbol{\lambda}_i$ by the shooting method. Therefore, a bounded positive $w(0)$ is chosen. Notice that the integrand of J^* has to be of class C^2 or greater. Therefore, w needs to satisfy $w(\beta) = w'(\beta) = w''(\beta) = 0$. Moreover, it is convenient if $w(r)$ is an even function. The just described conditions give the following polynomial function with the lowest power of r:

$$w(r) = \begin{cases} -\alpha(r^2 - \beta^2)^3, & \text{for} \quad |r| < \beta \\ 0, & \text{for} \quad |r| \ge \beta \end{cases} \tag{7}$$

which is shown in Fig. 1. The repulsive region corresponds to the ellipse of which the major axis along the longitude has double the length of the minor axis in the CW space. This is suitable for real situations because both longitudinal errors of thruster acceleration and position measurement are larger than the radial errors.

The extremization $\delta J^* = 0$ yields the Euler–Lagrange equations, that is,

$$\boldsymbol{\lambda}_i' = \begin{bmatrix} -3\lambda_i^{(3)} - \sum\limits_{j \ne i}^{n} \dfrac{\partial w(r_{ij})}{\partial x_i^{(1)}} \\ -\sum\limits_{j \ne i}^{n} \dfrac{\partial w(r_{ij})}{\partial x_i^{(2)}} \\ -\lambda_i^{(1)} + \lambda_i^{(4)} \\ -\lambda_i^{(2)} - 4\lambda_i^{(3)} \end{bmatrix}, \qquad u_i = -\lambda_i^{(4)} \tag{8}$$

$$\boldsymbol{\varphi}_i(t_f) = \mathbf{0}, \qquad \boldsymbol{\varphi}_i = \left(\lambda_i^{(1)} - \lambda_i^{(4)}, \lambda_i^{(2)} + \lambda_i^{(3)}\right)^T \tag{9}$$

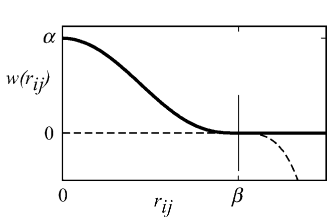

Fig. 1 Penalty function for near-miss avoidance.

Let $\boldsymbol{\mu} \in \mathcal{R}^{4n}$ denote $[\boldsymbol{\lambda}_1(0), \ldots, \boldsymbol{\lambda}_n(0)]^T$, that is, the initial costate vector. Before a shooting search is performed, $\boldsymbol{\mu}$ is found with $w(r_{ij}) = 0$. In this linear case, Eq. (8) is solved analytically as

$$\lambda_i^{(1)}(t) = \lambda_i^{(4)}(t) = \mu_i^{(1)} + 3\mu_i^{(2)} t$$
$$\lambda_i^{(2)}(t) = -\lambda_i^{(3)}(t) = \mu_i^{(2)} \tag{10}$$

The analytical solution given by Eq. (1) with the derived control function $u_i(t) = -\mu_i^{(1)} - 3\mu_i^{(2)} t$ is substituted for the terminal constraint Eq. (3), which yields

$$\mu_i^{(1)} = -2\left(3\epsilon_i^{(1)} + 2\epsilon_i^{(2)} t_f\right) \big/ \left(3t_f^2\right)$$
$$\mu_i^{(2)} = 2\left(2\epsilon_i^{(1)} + \epsilon_i^{(2)} t_f\right) \big/ \left(3t_f^3\right) \tag{11}$$

The optimum thrust acceleration is the linear function of t satisfying Eqs. (10) and (11) if no near miss happens.

If any near miss happens at the initial step, the following is a procedure for a sequential shooting search for an optimum $\boldsymbol{\mu}$. Let $\delta\alpha$ and $\delta\beta_\kappa$, for $\kappa = 0, 1, \ldots$, be small positive constants and β_C be a critical distance considered as a near miss. A sequence of shots $\boldsymbol{\mu}$ denotes $\boldsymbol{\mu}^m$, where m is a time of shooting iteration. Let $C(\alpha, \beta)$ denote that $\boldsymbol{\mu}^m$ converges by a shooting search with given α and β. The negation of the expression is denoted by $\neg$. The expression $x \leftarrow y$ is defined as substitute y for x. The iteration procedure is as follows:

$\alpha \leftarrow 0$, $\kappa \leftarrow 0$, $\beta \leftarrow \beta_C + \delta\beta_0$, and $\gamma \leftarrow \beta$. Iterate to perform the following five conditional commands after the shooting search.

P1) If $C(\alpha, \gamma)$, then $\alpha \leftarrow \alpha + \delta\alpha$.

P2) If $\neg C(\alpha, \gamma)$, then $\kappa \leftarrow \kappa + 1$ and $\gamma \leftarrow \gamma + \delta\beta_\kappa$.

P3) If $\gamma \ne \beta$ and $C(\alpha, \beta)$, then $\gamma \leftarrow \beta$.

P4) If $C(\alpha, \beta)$ and $\min r \ge \beta_C$, then the search for optimum $\boldsymbol{\mu}$ is terminated successfully.

P5) If $\neg C(\alpha, \varepsilon\beta)$ for arbitrary $\varepsilon > 0$, then the search for optimum $\boldsymbol{\mu}$ ends in failure.

Here, $\min r$ is a minimum instantaneous distance given by

$$\min r = \min\{\min\{r_{12}(t), \ldots, r_{ij}(t), \ldots, r_{n-1\,n}(t)\}\} \qquad t \in [0, t_f] \tag{12}$$

Therefore, $\min r > \beta_C$ in the terminating condition P4 shows that any near miss is avoided. When each shooting search ends in condition P1 or P3, the last $\boldsymbol{\mu}$ is regarded as an initial guess $\boldsymbol{\mu}^0$ for the next shooting search. When each shooting search ends in condition P2, $\boldsymbol{\mu}$ that satisfies the convergence condition latest is regarded as $\boldsymbol{\mu}^0$ for the next search. A shooting search corresponds to the Newton–Raphson method (see Ref. 12). Consider $\Lambda = [\boldsymbol{\psi}_1(t_f), \ldots, \boldsymbol{\psi}_n(t_f), \boldsymbol{\varphi}_1(t_f), \ldots, \boldsymbol{\varphi}_n(t_f)]^T \in \mathcal{R}^{4n}$ as a function of $\boldsymbol{\mu} \in \mathcal{R}^{4n}$. The $4n$-dimensional tangential plane of Λ at $\boldsymbol{\mu}^m$ intersects the $4n$-dimensional $\boldsymbol{\mu}$ plane in the $8n$-dimensional space. This intersection point is regarded as $\boldsymbol{\mu}^{m+1}$.

In principle, a shooting method of the boundary-value problem is repeated with fixing β and increasing α, following the conditional command P1. There is another way to expand w such that α is fixed and β is increased from zero. Changing β, however, means varying repulsive–interaction regions in the phase space, which is different from an increase of α corresponding to just strengthening the interaction. If α is not small, even a small change of β results in divergence of $\boldsymbol{\mu}^m$ because of the radical change of interaction dynamics. However, changing β can be the second best way while α is close to zero. In particular, this is considered to be effective if $\boldsymbol{\mu}^m$ neither converge nor diverge. The conditional commands P2 and P3 are, hence, included.

Implementation

The fuel-optimum injection into clustering satellites is now evaluated. Let the near-miss distance β_C be the unit distance 1 and the initial values be

$$\epsilon_i^{(1)} = \epsilon_i^{(2)} = 0, \qquad \epsilon_i^{(3)} = \sin\phi_i, \qquad \epsilon_i^{(4)} = \cos\phi_i$$

$$\phi_i = \pi i/3, \qquad i = 1, \ldots, 6$$

$$\epsilon_7^{(1)} = -t_f \epsilon_7^{(2)}, \qquad \epsilon_7^{(2)} = \pi/4, \qquad \epsilon_7^{(3)} = \epsilon_7^{(4)} = 0 \quad (13)$$

which corresponds to eccentricity separation for six satellites and eastward drift of the seventh satellite. Let t_f be 2π, that is, one period of the reference orbit. Without thrust, the satellites move along

$$\left[x_i^{(1)}(t), x_i^{(2)}(t)\right] = [\sin(\phi_i - t), \cos(\phi_i - t)], \qquad i = 1, \ldots, 6$$

$$\left[x_7^{(1)}(t), x_7^{(2)}(t)\right] = [-\pi/3, \pi(t - 2\pi)/4] \quad (14)$$

which puts all of the satellites in the following dangerous situations. The minimum distance of an initial cluster is equal to the near-miss distance β_C. Moreover, S_7 approaches S_4 and S_5. This is unrealistic because a cluster should be operated to leave a safety margin. These are, however, considered adequate for testing the procedure. The first step, that is, the fuel-optimum guidance without a penalty function, causes a collision between S_6 and S_7 because the satellites follow

$$\left[x_i^{(1)}(t_f), x_i^{(2)}(t_f)\right] = (\sin\phi_i, \cos\phi_i), \qquad i = 1, \ldots, 6$$

$$\left[x_7^{(1)}(t_f), x_7^{(2)}(t_f)\right] = (0, 1) \quad (15)$$

The penalty-function guidance is then necessary.

The sequence $\boldsymbol{\mu}^m$ is considered convergent if every component difference between the sequence is less than 10^{-5}. If $\boldsymbol{\mu}^m$ does not converge during 100 iterations, it is considered nonconvergent. The increments of β at the conditional command P2 are fixed as $\delta\beta_1 = 0.1\beta$, $\delta\beta_2 = 0.02\beta$, and $\delta\beta_\kappa = 0$ for $\kappa = 3, 4, \ldots$, respectively. Let $\delta\beta_0$ be given later. Each search is computed by a fourth-order Runge–Kutta integration with a 0.002π time step (1000th of a period), providing $\delta\alpha = 10^{-4}$ for $\alpha \le 0.01$ and 10^{-3} otherwise.

Figure 2 shows the result with $\delta\beta_0 = 0.04\beta_C$. The orbits of $(x_i^{(1)}, x_i^{(2)})$, $i = 1, \ldots, 7$, are drawn at the final command P4. The parameter α is increased to 0.571. The open and the filled circles represent the initial and the final points, respectively. The final state that S_7 is injected between S_1 and S_6 is obtained. For $t \ge t_f$, every satellite revolves once per orbital period around the origin with each final radius. All of the time-varying distances between satellites are graphed in Fig. 3 at the stage of $\alpha = 0.002$ and the final stage, respectively. Two bold curves show r_{56} and r_{67} in Figs. 3a and 3b. The horizontal broken line represents β_C. Although r_{12}, r_{45}, r_{56}, r_{67}, and r_{17} are judged near misses in the former stage, all of the satellites avoid any near miss in the latter stage corresponding to Fig. 2.

Next, the shooting searches are simulated for various $\delta\beta_0$. The consumed fuel and the α at the terminating condition P4 or P5 are evaluated in Table 1, where

$$\max|u| = \max\{\max\{|u_i(t)|, \ldots, |u_n(t)|\}\}, \qquad t \in [0, t_f] \quad (16)$$

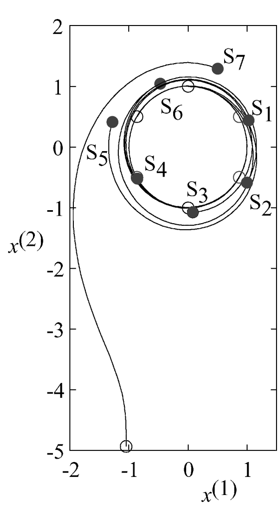

Fig. 2 Orbits of satellites obtained by penalty function with $\beta = 1.04$.

Table 1 Fuel consumed for various β margins

$\delta\beta_0$	$\min r$	$\mathrm{tot}\lvert\Delta v\rvert$	$\max\lvert u\rvert$	Final α
0.01	0.99719	0.947	0.1381	8.600
0.02	0.99924	0.966	0.1382	2.700
0.04	1	0.998	0.1376	0.571
0.08	1	1.05	0.1353	0.111
0.16	1	1.15	0.1305	0.021
0.32	1	1.21	0.1389	0.0033

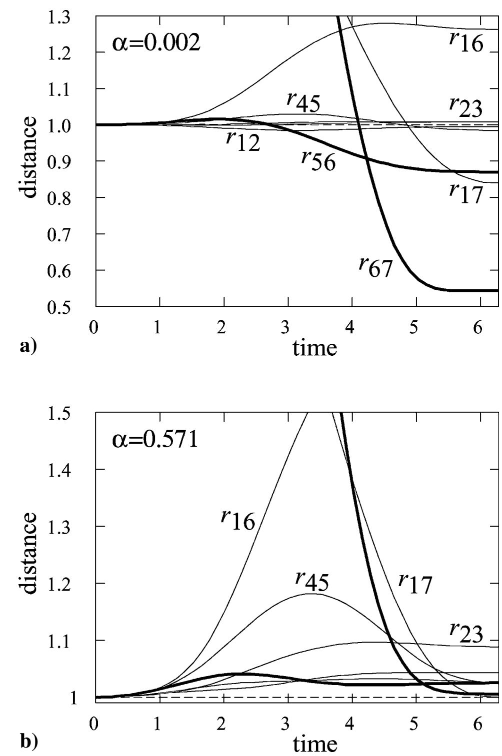

Fig. 3 Distances between respective satellites with $\beta = 1.04$.

For $\delta\beta_0 \le 0.02$, $\boldsymbol{\mu}^m$ diverges before $\min r \ge \beta_C$. Although the interruption condition P5 suggests trying searches for any ε, the simulation is abandoned without changing ε because α is already large. For $\delta\beta_0 \ge 0.04$, near-miss avoidance succeeds. A final α decreases and $\mathrm{tot}|\Delta v|$ increases with increasing $\delta\beta_0$. When it is assumed that each shooting search requires a constant duration, a final α is nearly proportional to the computation time. Thereby, a quick search is possible if additional fuel is used. Accordingly, $\delta\beta_0 \sim 0.1$ and $\delta\beta_0 \sim 0.2$ lead to $\mathrm{tot}|\Delta v| \sim 1.1$ with $\alpha \sim 0.1$ and $\mathrm{tot}|\Delta v| \sim 1.2$ with $\alpha \sim 0.01$, respectively. Table 1 implies that $\mathrm{tot}|\Delta v|$ by the exact optimum solution is expected as about 0.9. According to our defined α increment step, a 10% increase in β causes a 20% increase in fuel with about 200 time-shooting searches, and a 20% increase in β brings a 30% increase in fuel with about 100 searches. Therefore, guidance with moderate increases in fuel and reasonable shooting duration searches are constructed. However, improvement of the iterative process, for example systematic selection of $\delta\beta_\kappa$, is still necessary in future work.

Finally, the feasibility of the maximum instantaneous thrust $(2\max|u|) \sim 0.28$ is confirmed by fixing scales. The first example is cluster formation in geosynchronous orbit. A unit time is a period divided by 2π and then given by 13,713.4 s. A unit distance, that is, the near-miss distance, is assumed as 1000 m. Hence, a unit acceleration is interpreted as 5.31753×10^{-6} m/s^2, and then the maximum instantaneous thrust approximates 1.5 mN for 1000-kg mass. The second example is the clustering of low-Earth-orbit satellites. When a 400-km altitude orbit is assumed, a unit time is 884.1 s. If a unit distance is 500 m, a unit acceleration is given by 6.398×10^{-4} m/s^2, and then $(2\max|u|) \sim 18$ mN for 100-kg mass.

Conclusions

A fuel-minimization and near-miss avoidance control has been developed for clustered satellites. A penalty function of class C^2 with respect to the distances between satellites is imposed on the fuel-consumption index. The function is formulated as positive if any distance is less than a defined near-miss distance and as zero otherwise. Therefore, the Euler–Lagrange equations do not require a distinction between unconstrained and constrained modes. It is enough to solve just the two-point boundary-value problem even if unspecified multiple near misses occur between satellites. The proposed simulation procedure is to repeat shooting searches and to increase the height of the penalty function. The initial costate values are chosen under the assumption that there is no penalty function. Numerical simulation demonstrates that good control of clustered satellites is possible.

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